DHANAMANJURI UNIVERSITY

Examination- 2025 (June)

M.A/M.Sc. 2nd Semester

Name of Programme : **M.A/M. Sc. Mathematics**
Paper Type : **Theory**
Paper Code : **MAT-509**
Paper Title : **Complex Analysis-II**
Full Marks : **80**
Pass Marks : **32**

Duration: 3 Hours

The figures in the margin indicate full marks for the questions

Answer all the questions:

1. Answer any three of the following questions: $10 \times 3 = 30$

a) i) Discuss the existence of zeros of an entire function at infinity. 5

ii) For $\in \mathbb{N} \cup \{0\}$, prove that $|1 - E_p(z)| \leq |z|^{p+1}$ for all $|z| \leq 1$, where

$$E_p(z) = \begin{cases} 1 - z \; if \; p = 0 \\ (1-z)\exp\left(\sum_{k=1}^{p} \frac{z^k}{k}\right) \; if \; p \in \mathbb{N} \end{cases}$$ 5

b) i) State Weierstrass factorization theorem. Prove that $\sum_{n=1}^{\infty}\left|1 - E_{p_n}\left(\frac{z}{a_n}\right)\right|$ converges uniformly for every disk $|z| < r, r > 0$, where $< p_n >$ is a sequence of non-negative integers and $\{a_n\}_{n\geq 1}$ are non zero complex number such that $|a_n| \to \infty$ as $n \to \infty$. 5

ii) Represent $f(z) = \sin \pi z$ as its infinite product. 5

c) i) Prove that gamma function $\Gamma(z)$ is meromorphic. 3

ii) State and prove Mittag- Leffler theorem. 7

d) Define analytic continuation of a function f from a domain D to another domain D_1 and discuss the analytic continuation of the function $F(z) = \frac{1}{1-z}$. 10

Or

If $f(z) = \sum_{n=0}^{\infty} a_n z^n$ has a radius of convergence $R > 0$, then f must have at least one singularity on $|z| = R$ 10

2. **Answer any three of the following questions:** **$10 \times 3 = 30$**

a) State Gauss mean value theorem. Define Harmonic function and Harmonic conjugate. If $u(x, y)$ is harmonic in a simply connected domain D, prove that there exists an analytic function on D whose real part equals to $u(x, y)$. 1+2+7=10

b) Show that the function $u(x, y) = x + e^{-x} \cos y$ is harmonic and find its harmonic conjugate. State and prove the Schwarz Lemma. 5+5=10

c) If $u(z)$ is harmonic in a domain D and constant in the neighborhood of some point in D, then prove that $u(z)$ is constant throughout D. State and prove the Poisson integral formula for analytic function. 4+6=10

d) State and prove maximum principle for harmonic function. State and prove the Harnack's inequality. 4+6=10

3. **Answer any two of the following questions:** **$10 \times 2 = 20$**

a) Discuss different kinds of isolated singularity and their properties, give examples. State and prove Casorati-Weierstrass theorem. 4+6=10

b) Show that for any complex number c, $\sin z = c$ has infinitely many solutions. State and prove Riemann's theorem for removable singularity. 4+6=10

c) If $f(z)$ is non-constant entire function, then prove that $f(z)$ has essential singularity at ∞ if and only if $f(z)$ is a transcendental function. State and prove the open mapping theorem. 5+5=10

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